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ON THE AERODYNAMIC FORCES ACTING ON AN AIRFOIL SECTION WITH A PENETRABLE REGION

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Strict formulas for calculating the aerodynamic forces acting on a section of an airfoil with a penetrable region, through which a part of an external flow of an ideal incompressible fluid is sucked in, and on an airfoil section, from which a jet stream is blown out, in the case where these sections are in a steady-state flow without separation have been derived. Formulas for calculating the power expended for the realization of suction of a part of an external flow through an airfoil section and simultaneous blow-out of a jet stream from it are proposed.

To formulate and solve the boundary-value problems on the aerohydrodynamics of airfoil sections with penetrable regions through which suction or blow-out are realized, it is necessary to know the aerodynamic forces acting on these airfoil sections. Note that we take the phrase forces acting on an airfoil section to mean forces acting on a unit, in width, element of an airfoil section of infinite dimension. We will consider a steady-state flow without separation around an airfoil section.

In [1], where the blow-out of a flow of an incompressible ideal fluid was investigated, the main vector of the forces acting on an isolated airfoil section with a penetrable region, around which an incompressible ideal fluid flows, was determined by the Chaplygin formula

$$\overline{\mathbf{R}} = R_x - iR_y = \frac{\rho i}{2} \oint_{L_z} \left(\frac{dw}{dz}\right)^2 dz \, . \tag{1}$$

Integration over a closed airfoil outline L_z with a penetrable region MN was performed in a counter-clockwise direction (Fig. 1).

For an airfoil section with suction, the conjugate velocity $\overline{\mathbf{v}} = dw/dz$ of a flow in the neighborhood of a point at infinity in the region G_z can be represented, according to [1], in the form of the Laurent series

$$\frac{dw}{dz} = v_\infty + \frac{Q}{2\pi z} + \frac{\Gamma}{2\pi i z} + \sum_{k=2}^{\infty} c_k z^{-k} \, . \tag{2}$$

Here, $Q < 0$ for an airfoil section with suction and $Q > 0$ for an airfoil section with blow-out; the positive circulation Γ is performed in a counter-clockwise direction, and the direction of the abscissa axis is coincident with the direction of the velocity vector $\mathbf{v}_\infty$ of the incident flow. Since the flow considered has no distinctive features, we will pass from the integration over the outline L_z to a ring outline L_R of infinitely large radius. Substitution of expansion (2) into formula (1) gives

$$\overline{\mathbf{R}} = \frac{\rho i}{2} \oint_{L_z} \left(v_\infty + \frac{Q - i\Gamma}{2\pi z} + \sum_{k=2}^{\infty} c_k z^{-k} \right)^2 dz = \frac{\rho i}{2} \oint_{L_z} \left(v_\infty^2 + v_\infty \frac{Q - i\Gamma}{\pi z} + \sum_{k=2}^{\infty} c_k^* z^{-k} \right) dz \, ;$$

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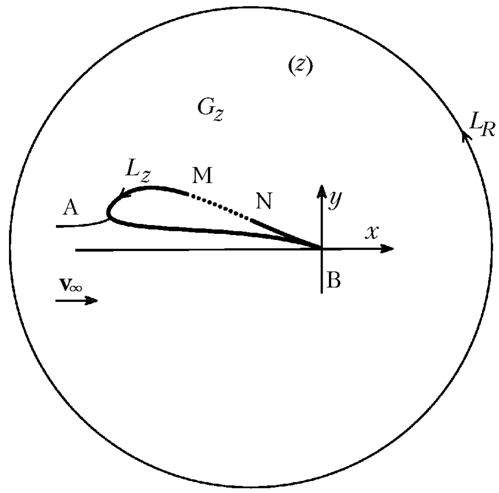

Fig. 1. Airfoil section in a fluid flow.

after simplifications, we obtain the expressions

$$R_x - iR_y = \frac{\rho i}{2} v_\infty \frac{Q - i\Gamma}{\pi} \ln \exp (2\pi i) = -\rho v_\infty (Q - i\Gamma) ,$$

from which it follows that

$$R_y = -\rho\Gamma v_\infty , \quad R_x = -\rho Q v_\infty . \tag{3}$$

The first formula of (3) represents the known Zhukovskii theorem on the aerodynamic lift, and the second formula is also a Zhukovskii formula given in [2].

It follows from formulas (3) that, for an airfoil around which a steady-state flow of an incompressible ideal fluid without separation flows, the aerodynamic lift R_y and the drag force R_x are directly proportional to the circulation of the flow velocity and the flow rate respectively. Thus, if the rates of flow around airfoil sections with suction are equal, the drag forces of these sections will be equal independently of the location of penetrable regions and the angle at which the suction is realized. Formula (1) was derived with assumptions from [3, pp. 192–195], which are true only for impenetrable airfoil sections; for example, it was assumed that the flow velocity is directed, everywhere, along the tangent line to the outline L_z of an airfoil. Taking into account the foregoing, we will perform strict derivation of formulas (1) and (3) for calculating the aerodynamic forces acting on an airfoil section with a penetrable region. Moreover, we will present and substantiate formulas for calculating the aerodynamic forces acting on an airfoil section, from which a jet stream with a total pressure and a density different from those of the external flow is blown out. Finally, we will present formulas for calculating the power expended for the realization of suction of a flow of an incompressible ideal fluid through an airfoil section and simultaneous blow-out of a jet stream from it.

Derivation of a Formula for an Airfoil Section with Suction of an External Flow. We will consider the region ds of the outline L_z of an airfoil with suction. The force acting on this region is equal, in accordance with the momentum-conservation law, to

$$d\mathbf{R} = -[p\mathbf{n} + \rho (\mathbf{v}, \mathbf{n}) \ \mathbf{v}]\, ds ,$$

where $\mathbf{n}$ is the vector of the normal to the outline (Fig. 2). Having performed integration over the outline L_z, we will find the force vector $\mathbf{R}$:

$$\mathbf{R} = \oint_{L_z} d\mathbf{R} = \oint_{L_z} [-p_0\mathbf{n} + \rho (\mathbf{v}, \mathbf{v})\, \mathbf{n}/2 - \rho (\mathbf{v}, \mathbf{n})\, \mathbf{v}]\, ds . \tag{4}$$

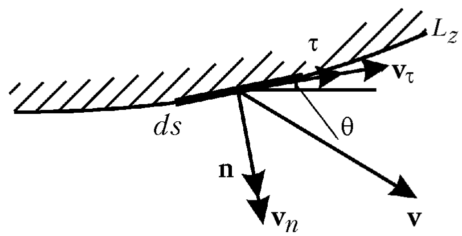

Fig. 2. Region of the airfoil outline L_z.

In the case of suction of a flow, p_0 = const throughout the flow region.

Let us pass to complex variables. Note that

$$\boldsymbol{\tau} = \exp(i\theta)\,, \quad \mathbf{n} = -\,i\exp(i\theta)\,, \quad d\mathbf{z} = \exp(i\theta)\,ds\,, \quad ds = \exp(i\theta)\,\overline{d\mathbf{z}} = \exp(-\,i\theta)\,d\mathbf{z}\,,$$

$$\mathbf{v}_\tau = v_\tau \exp(i\theta)\,, \quad \mathbf{v}_n = -\,iv_n \exp(i\theta)\,, \quad \mathbf{v} = \mathbf{v}_\tau + \mathbf{v}_n = (v_\tau - iv_n)\exp(i\theta)\,.$$

Then

$$(\mathbf{v}, \mathbf{v}) = |v|^2 = v_\tau^2 + v_n^2 = (\mathbf{v}_\tau^2 - \mathbf{v}_n^2)\exp(-\,2i\theta)\,, \quad (\mathbf{v}, \mathbf{n}) = v_n = \mathbf{v}_n i\exp(-\,i\theta)\,.$$

Substitution of these variables into (4) gives

$$\mathbf{R} = \oint_{L_z} [p_0 i\exp(i\theta - i\theta)]\,dz - \oint_{L_z} [\rho i\,(\mathbf{v}_\tau^2 - \mathbf{v}_n^2)/2 + \rho\mathbf{v}_n i\,(\mathbf{v}_\tau + \mathbf{v}_n)]\,\overline{dz}\,.$$

After simple rearrangements, we obtain

$$\mathbf{R} = \oint_{L_z} \left[ip_0 dz - \frac{\rho i}{2}(\mathbf{v}_\tau + \mathbf{v}_n)^2\,\overline{dz} \right] = \oint_{L_z} \left[ip_0 dz - \frac{\rho i}{2}\overline{\left(\frac{dw}{dz}\right)^2}\,\overline{dz} \right]. \tag{5}$$

Since the integral of the constant quantity p_0 over the close curve L_z is equal to zero, we will obtain, having performed complex conjugation, the following formula for the resultant force acting on an airfoil section with a penetrable region:

$$\overline{\mathbf{R}} = \frac{\rho i}{2}\oint_{L_z}\left(\frac{dw}{dz}\right)^2 dz\,,$$

which is entirely identical to the Chaplygin formula (1). Consequently, formula (1) can be used for calculating airfoil sections with a penetrable region, as was done by V. V. Golubev in [1].

The aerodynamic-lift coefficient c_y and the drag coefficient c_x, related to the chord b of the section and the velocity v_∞ of the incident flow, will be equal, according to (3), to

$$c_y = -\,2\gamma\,, \quad c_x = -\,2q\,, \quad \gamma = \Gamma/bv_\infty\,, \quad q = Q/bv_\infty\,. \tag{6}$$

Derivation of a Formula for an Airfoil Section with Blow-Out of a Jet Stream to the External Flow. As was shown above, formulas (1) and (3) can be used for penetrable airfoil sections with suction of a flow. However, in the case where a jet is blown out of an airfoil section, the density and total pressure of the liquid in this jet can differ from those in the external flow. In this case, formulas (1) and (3) will be incorrect. This is explained by the fact that, at the interface between the external flow and the jet, the tangential component of the flow velocity will be disrupted, with the result that the function of the complex-conjugate velocity will become piecewise-analytical. At the same time, formula (5) remains true also in the case where the densities and total pressures of the jet and the external

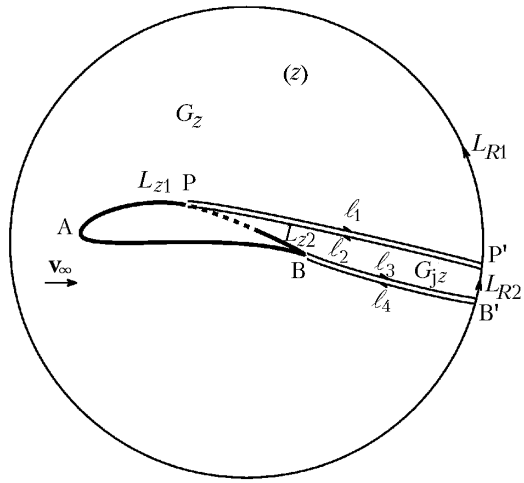

Fig. 3. Airfoil section with jet blow-out.

flow are different, because this formula was obtained with general assumptions and the indicated conditions were taken into account at a later time. Using (5), we will obtain a formula for the aerodynamic forces acting on airfoil sections with blow-out of a jet stream.

Let the density ρ_j and the total pressure p_{j0} of a jet blown out of an airfoil section differ from the density ρ and the total pressure p_0 of an external flow. Therefore, the tangential velocity components PP$'$ and BB$'$ of a downflow will be disrupted (Fig. 3). The points B and P divide the outline L_z of the airfoil section into two parts: the part in contact with the external flow L_{z1} and the part in contact with the blown-out jet L_{z2}. For simplicity, we will investigate the case of distributed blow-out where a penetrable region is located on L_{z2} (the case of slot blow-out can be investigated analogously). Let us consider two fluid volumes bounded by the closed outlines $L_{z1} \cup l_1 \cup L_{R1} \cup l_4$ and $L_{z2} \cup l_3 \cup L_{R2} \cup l_2$, where L_{R1} and L_{R2} are the arcs of a circle of infinitely large radius.

The force acting on the airfoil section will be equal to

$$\mathbf{R} = \oint\limits_{L_{z1} \cup L_{z2}} \left[i p_0 dz - \frac{\rho i}{2} \left(\frac{dw}{dz} \right)^2 \overline{dz} \right] = \oint\limits_{L_{R1} \cup L_{R2}} \left[i p_0 dz - \frac{\rho i}{2} \left(\frac{dw}{dz} \right)^2 \overline{dz} \right]. \tag{7}$$

The functions $p_0(z)$ and $\rho(z)$ involved in the integration elements are equal to p_0 and ρ for the external flow (G_z) and ρ_{j0} and ρ_j for the jet (G_{jz}). The complex-conjugate velocity of the flow in the regions G_z and G_{jz} is equal to

$$\left.\frac{dw}{dz}\right|_{G_z} = v_\infty + \frac{\tilde{Q} - i\tilde{\Gamma}}{2\pi z} + \sum_{k=2}^{\infty} c_k z^{-k}, \quad \left.\frac{dw}{dz}\right|_{G_{jz}} = v_{j\infty} + \sum_{k=1}^{\infty} c_{jk} z^{-k}, \tag{8}$$

where $\tilde{Q}/2\pi$ and $-\tilde{\Gamma}/2\pi$ are the real and imaginary parts of the expansion coefficient of the function dw/dz at z^{-1} in the neighborhood of a point at infinity in the region G_z. Note that, when separations are absent in the external flow (in the case where $p_0 = p_{j0}$ and $\rho_j = \rho$), the quantity $\tilde{Q}$ becomes equal to the flow rate Q and the quantity $\tilde{\Gamma}$ becomes equal to the circulation Γ of the flow velocity over the outline of the airfoil section.

Substitution of (8) into (7) gives

$$\mathbf{R} = i\,(p_{j0} - p_0)\, ih_\infty - \frac{i}{2} \left[\overline{\rho v_\infty^2 (-ih_\infty) + \rho v_\infty (\tilde{Q} - i\tilde{\Gamma})\, 2i + \rho_j v_{j\infty}^2 i h_\infty} \right].$$

Having performed complex conjugation with the use of the Bernoulli integral, we obtain

$$\mathbf{R} = -h_\infty (\rho_j v_{j\infty}^2 - \rho v_\infty^2) - \rho v_\infty \tilde{Q} - i \rho v_\infty \tilde{\Gamma}\,.$$

Here, $h_\infty = Q/v_{j\infty} = \tilde{Q}/v_\infty$. Unfortunately, we failed to find a simple relation between $\tilde{\Gamma}$ and Γ. After simple rearrangements, we obtain

$$R_y = -\rho v_\infty \tilde{\Gamma}\,, \quad R_x = -\rho_j v_{j\infty} Q = -\sqrt{\rho\rho_j\,(1+\mu)}\; v_\infty Q\,, \tag{9}$$

where

$$\mu \equiv \frac{2\,(p_{j0}-p_0)}{\rho v_\infty^2} = \frac{\rho_j v_{j\infty}^2}{\rho v_\infty^2} - 1 \tag{10}$$

characterizes the energy of the blown-out jet.

Formulas analogous to (9) were obtained by N. F. Vorob'ev in [4] for the "section of an airfoil, the lower side of which is formed by a system of guide vanes around which a flow incoming from the inner space of the airfoil and forming a jet in the external flow flows."

The coefficients of drag (c_x) and aerodynamic lift (c_y) will be equal, in view of (9), to

$$c_y = -2\tilde{\gamma}\,, \quad c_x = -2q\sqrt{\frac{\rho_j\,(1+\mu)}{\rho}}\;. \tag{11}$$

Airfoil Section with Suction and Blow-Out. Calculation of the Power Expended for These Processes. It will be assumed that, in the case of an airfoil section with suction, the fluid sucked from the external flow is not accumulated but is blown out to a flow (though not necessarily on the airfoil section). Analogously, for an airfoil section with blow-out, the suction of a fluid from an external flow should be organized. If $\rho_j = \rho$, the resultant coefficient of drag c_x will comprise the sum of the coefficient of drag c_{xs} (6), arising as a result of the suction, and the coefficient of thrust c_{xj} (11), arising as a result of the blow-out:

$$c_x = c_{xs} + c_{xj} = 2q\,(1-\sqrt{1+\mu})\,.$$

The coefficient $c_x < 0$ at $\mu > 0$, which corresponds to the case of existence of a propulsive force. We will express the power expended for the above-indicated suction–blow-out in terms of the equivalent drag coefficient c_{xp}. This coefficient is determined from the formula presented in [5]:

$$c_{xp} = \eta_{en}\frac{P}{b v_\infty \rho v_\infty^2/2}\,. \tag{12}$$

It will be assumed that the fluid of an external flow entering a blowing power plant has a total pressure p_0 and the outflowing fluid has a total pressure p_{j0}. In this case, the power of this plant will be equal to

$$P = (p_{j0}-p_0)\,Q/\eta_p\,.$$

For the coefficient c_{xp}, we obtain, using (12), the following expression:

$$c_{xp} = \frac{\eta_{en}}{\eta_p}\,\frac{(p_{j0}-p_0)}{\rho v_\infty^2/2}\,\frac{Q}{b v_\infty} = \frac{\eta_{en}}{\eta_p}\,\mu q\,.$$

In the particular case where the efficiencies of an engine and a power plant are equal ($\eta_{en} = \eta_p$), we obtain, using (10), the simple formula

$$c_{xp} = \mu q\,.$$

It should be noted that $c_{xp} + c_x > 0$ at any μ. As was expected, the power expended for the suction–blow-out is larger than the power obtained from this effect, and the sum $c_{xp} + c_x$ increases with increase in the energy of the jet blown out.

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NOTATION

b, chord of an airfoil section; c_k, $\overset{*}{c}_k$, coefficients in the Laurent series; c_x, drag coefficient; c_{xj}, drag coefficient in the case of blow-out; c_{xp}, expenditure of energy; c_{xs}, drag coefficient in the case of suction; c_y, lift coefficient; G_{jz}, region of a jet; G_z, region of an external flow; h_∞, width of a jet at infinity; $L_R = L_{R1} + L_{R2}$, circle of infinitely large radius; $L_z = L_{z1} \cup L_{z2}$, airfoil outline; $l_1 - l_4$, downflow lines; $\mathbf{n}$, vector of the normal to the outline of an airfoil; p, pressure; P, power of a power plant; p_{j0}, total pressure of a blown-out jet; p_0, total pressure; Q, rate of a total flow passing through an airfoil section; q, dimensionless flow rate; $\mathbf{R}$, vector of the resultant force; R_x, drag force; R_y, aerodynamic lift; s, arc abscissa of the outline of an airfoil; $\mathbf{v}$, velocity vector; $v_{j\infty}$, velocity of a jet at infinity; v_∞, velocity of an external flow at infinity; w, complex potential of a flow; $z = x + iy$, coordinate in the physical plane; Γ, circulation of the flow velocity over the outline of an airfoil; γ, dimensionless velocity circulation; η_a, efficiency of an engine; η_p, efficiency of a suction–blow-out setup; θ, angle of inclination of the tangent to the outline of an airfoil section; μ, dimensionless energy of a blown-out jet; ρ, density of an external flow; ρ_j, density of a blown-out jet; τ, vector of the tangent to the outline of an airfoil section. Subscripts: a, engine; j, blown-out jet; k, variable of summation in an infinite Laurent series; n, normal component of a vector (projection to the normal $\mathbf{n}$); p, suction — blow-out setup (pump); s, suction; x and y, projections of a vector on the x and y axes, respectively; z, physical density; τ, tangential component of a vector; ∞, characteristics at infinity; overscribed bar, complex conjugation.

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